The Role of $\Lambda(1405)$ in Kaon-Proton Interactions

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ABSTRACT

S-wave K^-p scattering into various channels near threshold are analyzed in heavy-baryon chiral perturbation theory with $\Lambda(1405)$ introduced as an independent field. This is the approach that predicted the critical density $2 \lesssim \rho_c/\rho_0 \lesssim 3$ for negatively charged kaon condensation. We show that chiral perturbation expansion treating the $\Lambda(1405)$ as elementary is consistent with *all* threshold data including a double-charge-exchange process suppressed at leading order of chiral expansion in the absence of the $\Lambda(1405)$. We also discuss S-wave K^+p scattering phase shifts at low energy.

1 Introduction

It has been shown [1, 2] that chiral perturbation theory to next-to-next-to-leading order ($\mathcal{O}(Q^3)$) with counterterms fixed by KN scattering lengths [3] and kaonic atom data[4] predicts that kaon condensation takes place in dense, compact-star matter at a density $2 \lesssim \rho_c/\rho_0 \lesssim 4$. It was found there that while the $\Lambda(1405)$ treated as an "elementary" field [#1] plays a crucial role in K^-p scattering and also in kaonic atoms, it has very little influence on kaon condensation. This is mainly because kaon condensation occurs in initially dense neutron matter and furthermore at a kinematical regime which is far from the pole position of the $\Lambda(1405)$. In fact, the prediction of the kaon condensation threshold was found to be surprisingly robust not only against the influence of the $\Lambda(1405)$ but also against uncertainties in the parameters of the theory.

An interesting – and highly pertinent – question was raised by Weise in his recent seminar [6]: Is the description treating the $\Lambda(1405)$ as "elementary" consistent with all low-energy KN data (e.g. the double-charge-exchange process $K^-p \rightarrow \pi^+\Sigma^-$)? Weise has proposed a dynamical coupled-channel treatment of the $\Lambda(1405)$ with a potential of range $\sim (400\text{MeV})^{-1}$ based on a chiral Lagrangian at $\mathcal{O}(Q^2)$. This dynamical model is found to successfully describe all of the available low-energy KN data, in particular, the observation that the OZI-suppressed process $K^-p \rightarrow \pi^+\Sigma^-$ has a lot bigger cross section than the process $K^-p \rightarrow \pi^-\Sigma^+$ allowed at the tree level.

An immediate question is: Would this dynamical picture not give a different prediction for kaon condensation?

We shall show in this letter that once the $\Lambda(1405)$ is implemented as an "elementary" field with the parameters of the chiral Lagrangian determined by the S-wave scattering lengths, the leading order ($\mathcal{O}(Q)$) chiral Lagrangian can explain most of the branching ratios and low-energy phase shifts within our framework. This is not so surprising if one recalls that in the Skyrme model, the $\Lambda(1405)$ is a bound state of a kaon and a soliton so that it cannot be described by a sum of finite series in chiral perturbation theory. A natural approach in the context of chiral perturbation theory to such a bound state is to solve a Lippman-Schwinger equation with a potential generated by chiral

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[#1] Whether or not the notion of $\Lambda(1405)$ as an elementary field can be justified in chiral perturbation theory for KN scattering is certainly an open question in view of the proximity of the $\Lambda(1405)$ pole to the KN threshold [5]. What we did in [1, 2] and what we shall do in this paper is to assume that the $\Lambda(1405)$ is elementary and to see how far one can go in explaining various aspects of low-energy kaon-nucleon and kaon-nuclear interactions.

perturbation theory using "irreducible" graphs in the sense of Weinberg or if one is sufficiently far from the singularity of the bound state as in the case of kaon condensation, to introduce it as an elementary field as we do here. The former corresponds to Weise's approach and the latter to ours. Our assertion in [2] was that we are using chiral Lagrangians in a region far away from the pole position of the bound state, our approach should be fully justified.

2 K^-p Scattering

We start by writing down the effective chiral Lagrangian that we shall use in the calculation. Let the characteristic energy/momentum scale that we are interested in be denoted Q. The standard chiral counting orders the physical amplitudes as a power series in Q, say, Q^ν, with ν an integer. To leading order, the kaon-nucleon amplitude goes as $\mathcal{O}(Q^1)$, to next order as $\mathcal{O}(Q^2)$ involving no loops. Following Jenkins and Manohar[7], we denote the velocity-dependent octet baryon fields B_v, the octet meson fields $\exp(i\pi_a T_a/f) \equiv \xi$, the velocity four-vector v_μ and the spin operator $S_v^\mu(v \cdot S_v = 0, S_v^2 = -3/4)$, the vector current $V_\mu = [\xi^\dagger, \partial_\mu \xi]/2$ and the axial-vector current $A_\mu = i\{\xi^\dagger, \partial_\mu \xi\}/2$, and write the Lagrangian density to order Q^2, relevant for the low-energy scattering, as

$$
\begin{aligned}
\mathcal{L} =\ & \text{Tr}\bar{B}_v(iv \cdot D)B_v + 2D\text{Tr}\bar{B}_v S_v^\mu \{A_\mu, B_v\} + 2F\text{Tr}\bar{B}_v S_v^\mu [A_\mu, B_v] \\
& + (\sqrt{2}g_{\Lambda^\star}\bar{\Lambda}_v^\star \text{Tr}(v \cdot A B_v) + h.c.) \\
& + a_1 \text{Tr}\bar{B}_v \mathcal{M}_+ B_v + a_2 \text{Tr}\bar{B}_v B_v \mathcal{M}_+ + a_3 \text{Tr}\bar{B}_v B_v \text{Tr}\mathcal{M}_+ \\
& + d_1 \text{Tr}\bar{B}_v A^2 B_v + d_2 \text{Tr}\bar{B}_v(v \cdot A)^2 B_v + d_3 \text{Tr}\bar{B}_v B_v A^2 + d_4 \text{Tr}\bar{B}_v B_v(v \cdot A)^2 \\
& + d_5 \text{Tr}\bar{B}_v B_v \text{Tr}A^2 + d_6 \text{Tr}\bar{B}_v B_v \text{Tr}(v \cdot A)^2 \\
& + d_7 \text{Tr}\bar{B}_v A_\mu \text{Tr}B_v A^\mu + d_8 \text{Tr}\bar{B}_v v \cdot A \text{Tr}B_v v \cdot A \\
& + d_9 \text{Tr}\bar{B}_v A_\mu B_v A^\mu + d_{10} \text{Tr}\bar{B}_v v \cdot A B_v v \cdot A
\end{aligned}
\tag{1}
$$

where D=0.81 and F=0.44, and the covariant derivative $\mathcal{D}_\mu$ for baryon fields is defined by

$$
\mathcal{D}_\mu B_v = \partial B_v + [V_\mu, B_v].
\tag{2}
$$

and $\mathcal{M}_+ \equiv \xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi^\dagger$ with $\mathcal{M} = diag(m_u, m_d, m_s)$.

In (1), the $\Lambda(1405)$ – denoted $\Lambda^\star$ – is introduced as a matter field on the same footing with the usual octet baryons. The $\Lambda^\star KN$ coupling constant, $g_{\Lambda^\star}^2 = 0.15$, is fixed to give the empirical decay width $\Gamma_{\Lambda^\star} = 50$ MeV through $\Lambda^\star \to \pi\Sigma$ with $m_{\Lambda^\star} = 1405$ MeV [1]. The explicit symmetry breaking a_i terms can be determined by the baryon mass splitting and the πN sigma term. Here we use the results of Kaplan and Nelson[8],

$$a_1 = -0.28, \quad a_2 = 0.56, \quad a_3 = -1.1$$

$$m_u = 6 \text{ MeV}, \quad m_d = 12 \text{ MeV}, \quad m_s = 240 \text{ MeV}. \tag{3}$$

These parameters give the sigma term

$$\Sigma_{KN} = -\frac{1}{2}(m_u + m_s)(a_1 + 2a_2 + 4a_3) \simeq 438 \text{ MeV}. \tag{4}$$

Given the large error bar (± 0.3) in a_3, this Σ_{KN} is consistent with the recent lattice calculations of Dong & Liu[9] (see also Fukugita et al. [10]),

$$\Sigma_{KN} = \frac{(m_u + m_s)\langle N|(\bar{u}u + \bar{s}s)|N\rangle}{(m_u + m_d)\langle N|(\bar{u}u + \bar{d}d)|N\rangle}\Sigma_{\pi N} \simeq 450 \pm 30 \text{ MeV}. \tag{5}$$

For S-wave scattering, there is no distinction between A^2 and $(v \cdot A)^2$ in (1). So the combination

$$\bar{d}_i = d_i + d_{i+1} \quad (i : \text{odd number}) \tag{6}$$

enters in the scattering amplitudes. This means that there are five independent parameters at $\mathcal{O}(Q^2)$ for S-wave KN scattering. These parameters need to be fixed for the equation of state of nuclear star with kaon condensation [11]. The transition matrix elements for various channels are summarized in Appendix.

Now requiring consistency with the experimental KN scattering lengths[3][#2],

$$a_0^{K^+p} = -0.31fm, \quad a_0^{K^-p} = -0.67 + i0.63fm$$

$$a_0^{K^+n} = -0.20fm, \quad a_0^{K^-n} = +0.37 + i0.57fm, \tag{7}$$

we get two constraints on $\bar{d}_i$ at $\mathcal{O}(Q^2)$[1],

$$(\bar{d}_s - \bar{d}_v)_{emp} \approx (0.05 - 0.06)fm$$

$$(\bar{d}_s + \bar{d}_v)_{emp} \approx 0.13fm \tag{8}$$

[#2]We are not concerned with fine-tuning of the parameters and hence no errors will be quoted.

with the isoscalar constants d_s and the isovector constants d_v defined by

$$\begin{aligned}
\bar{d}_s &= -\frac{1}{2M_K^2}(m_u + m_s)(a_1 + 2a_2 + 4a_3) + \frac{1}{4}(\bar{d}_1 + \bar{2}d_3 + 4\bar{d}_5 + \bar{d}_7) \\
\bar{d}_v &= -\frac{1}{2M_K^2}(m_u + m_s)\, a_1 + \frac{1}{4}(\bar{d}_1 + \bar{d}_7).
\end{aligned} \tag{9}$$

We are therefore left with three independent parameters to be determined from other experimental data.

3 Threshold Branching Ratios

Once the constants are fixed from some experiments, we could then make predictions for other physical quantities in low-energy KN scattering. In particular, the following threshold branching ratios are of particular interest [6]:

$$\begin{aligned}
\gamma &= \frac{|\mathcal{T}_{\pi^+\Sigma^-}|^2}{|\mathcal{T}_{\pi^-\Sigma^+}|^2} \\
R_c &= \frac{\sum_{i=\pi^\pm\Sigma^\mp} |\mathcal{T}_i|^2}{\sum_{j=\pi^0\Lambda,\pi^0\Sigma^0,\pi^\pm\Sigma^\mp} |\mathcal{T}_j|^2} \\
R_n &= \frac{|\mathcal{T}_{\pi^0\Lambda}|^2}{\sum_{i=\pi^0\Lambda,\pi^0\Sigma^0} |\mathcal{T}_i|^2}
\end{aligned} \tag{10}$$

The empirical values are [12]

$$\gamma^{exp} = 2.36 \pm 0.04, \quad R_c^{exp} = 0.664 \pm 0.011, \quad R_n^{exp} = 0.19 \pm 0.02. \tag{11}$$

Since we still have three constants left unfixed, we cannot compare theory directly with experiments for these quantities. However for the chiral perturbation approach to be viable, the leading $\mathcal{O}(Q^1)$ chiral order which involves no unknown counterterms should dominate. In other words, higher-order terms should be suppressed according to the counting rule, $(Q/4\pi f)^\nu$. To verify this, we calculate the branching ratios with the scattering amplitudes computed at $\mathcal{O}(Q^1)$. The results are summarized in Table 1. The numerical results are

$$\gamma = 1.93, \quad R_c = 0.64, \quad R_n = 0.11. \tag{12}$$

Here only the Weinberg-Tomozawa term and the $\Lambda^\star$ contribution in leading order are taken into account. Clearly the leading tree contributions play a dominant role for γ and R_c while R_n apparently requires some higher order corrections. Note that without the $\Lambda^\star$, the transition to $\pi^+\Sigma^-$

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channel	$f^2 \mathcal{T}_{\nu=1}$	$f^2 \mathcal{T}_{\Lambda^\star}$
$\pi^- \Sigma^+$	$(2M_K + m_B - m_{\Sigma^+})/4$	$g(m_{\Sigma^+})$
$\pi^+ \Sigma^-$	$-$	$g(m_{\Sigma^-})$
$K^- p$	M_K	$g(m_p)$
$\bar{K}^0 n$	$M_K/2$	$g(m_n)$
$\pi^0 \Sigma^0$	$(2M_K + m_B - m_{\Sigma^0})/8$	$g(m_{\Sigma^0})$
$\pi^0 \Lambda$	$\sqrt{3}(2M_K + m_B - m_\Lambda)/8$	$-$
$\eta \Sigma^0$	$\sqrt{3}(2M_K + m_B - m_{\Sigma^0})/8$	$-$
$\eta \Lambda$	$3(2M_K + m_B - m_\Lambda)/8$	$g(m_\Lambda)$
$K^+ \Xi^-$	$-$	$g(m_{\Xi^-})$
$K^0 \Xi^0$	$-$	$g(m_{\Xi^0})$

$$g(m) = -g_{\Lambda^\star}^2 \frac{M_K(M_K + m_B - m)}{m_B + M_K - m_{\Lambda^\star}}$$

Table 1: Leading-order contributions to $K^- p$ scattering

$\bar{d}_1 + \bar{d}_7$	$\bar{d}_3 + \bar{d}_7$	$\bar{d}_7 + \bar{d}_9$	γ	R_c	R_n
0.039	-3.35	-5.90	2.36	0.63	0.19

Table 2: Best-fit parameters iin fm for the branching ratios

would be suppressed at the leading order, so we would have $\gamma = 0$. Furthermore the enhancement of the $\pi^+\Sigma^-$ channel over the $\pi^-\Sigma^+$ channel is principally due to $\Lambda^\star$: while only the $\Lambda^\star$ term contributes to the $\pi^+\Sigma^-$ channel, both the Weinberg-Tomozawa term and the $\Lambda^\star$ term contribute to the channel $\pi^-\Sigma^+$ but with an opposite sign, giving rise to the enhancement of the ratio γ.

From the point of view of chiral perturbation theory for kaon condensation, the most meaningful outcome of the present exercise is that we are now able to determine the three parameter combinations that appear at $\mathcal{O}(Q^2)$. This determination would allow us to calculate to $\mathcal{O}(Q^2)$ the equation of state needed for describing the properties of the kaon condensed state. The best-fit parameters using $(\bar{d}_s - \bar{d}_v)_{emp} = 0.055 fm$ are given in Table 2.

4   S-Wave Phase Shift for K^+p Scattering

While the $\Lambda(1405)$ contributes unimportantly to the S-wave phase shifts for K^+p scattering (since it enters in the crossed term with a large energy denominator), it is however important to check whether or not the chiral perturbation approach to kaon-nuclear interactions with the large KN sigma term, $\Sigma_{KN} \approx 438$ MeV, is consistent with the well-measured K^+p phase shifts [15, 16]. We have computed the phase shifts to $\mathcal{O}(Q^2)$ chiral order for which there are no unknown constants once the scattering lengths are fit. The results are given in Table 3 and and Fig. 1. Although there are no experimental data to compare with for $P_{lab} < 145$ MeV, we see that the $\mathcal{O}(Q^2)$ chiral perturbation theory is consistent with the low-energy part of the phase shifts up to $P_{lab} \sim m_\pi$. The deviation seen at $P_{lab} \gtrsim 145$ MeV must clearly be due to higher chiral order terms. This will be checked in a future publication at $\mathcal{O}(Q^3)$ and beyond using the result of Ref.[2].

P_{lab}	$\omega_{K,cm}$	$W.\ Cameron$[15]	$R.A.\ Burnstein$[16]	$Our\ Result$
145	504	-8.2 ± 0.9	$-$	-9.05
175	508	-10.2 ± 0.3	$-$	-11.2
178	509	$-$	-10.1 ± 0.8	-11.4
205	513	-11.4 ± 0.4	$-$	-13.4
235	518	-13.3 ± 0.4	$-$	-15.8
265	525	-15.0 ± 0.2	-16.0 ± 0.4	-18.3

Table 3: Comparison of the predicted phase shifts (in deg) with experimental data. P_{lab} and $\omega_{K,cm}$ are given in MeV.
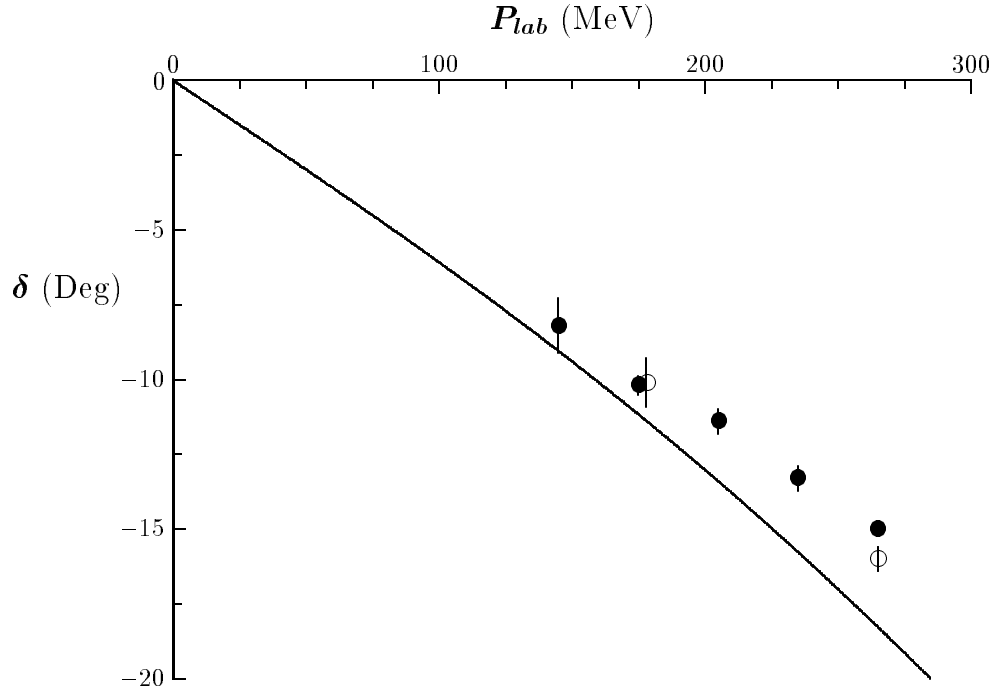
Figure 1: The S-wave phase shifts (in deg) for K^+p scattering. The filled and empty circles correspond to the data of [15] and [16], respectively.

5 Discussion

We have shown that chiral perturbation theory with the $\Lambda(1405)$ introduced as an elementary matter field can satisfactorily describe low-energy S-wave $K^{\pm}p$ scattering. In particular, the OZI-suppressed double-charge-exchange process $K^- p \to \pi^+ \Sigma^-$ is found to be enhanced relative to the OZI-allowed process $K^- p \to \pi^- \Sigma^+$ when $\Lambda(1405)$ is introduced. We have also shown that a large KN sigma term, $\Sigma_{KN} \approx 430$ MeV, is consistent with the experimental phase shifts for $K^+ p$ scattering at low energy, $P_{lab} \lesssim m_\pi$. Our results provide evidence that the $\Lambda(1405)$ as an elementary field is a phenomenologically viable concept, in a way that resembles the Δ in the nonstrange sector.

Finally we reiterate the conclusion of [2] that the $\Lambda(1405)$ interpolating as an elementary matter field plays a negligible role in kaon condensation with the subtleties associated with the threshold properties of KN scattering affecting little the condensation phenomenon.

Appendix : Transition Amplitudes

The transition matrix elements for various channels of threshold $K^- p$ scattering are

$$
\begin{aligned}
\mathcal{T}(K^- p \to \pi^- \Sigma^+) \;=\; & \frac{1}{4f^2}(2M_K + m_p - m_{\Sigma^+}) - \frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_{\Sigma^+})}{m_p + M_K - m_{\Lambda^\star}} \\
& -\frac{1}{2f^2}a_2(2m_u + m_d + m_s) \\
& +\frac{1}{2f^2}(\bar{d}_3 + \bar{d}_7)M_K(M_K + m_p - m_{\Sigma^+})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}(K^-p \to \pi^+\Sigma^-) &= -\frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_{\Sigma^-})}{m_p + M_K - m_{\Lambda^\star}} \\
&\quad +\frac{1}{2f^2}(\bar{d}_7 + \bar{d}_9)M_K(M_K + m_p - m_{\Sigma^-}) \\[2mm]
\mathcal{T}(K^-p \to K^-p) &= \frac{1}{f^2}M_K - \frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K^2}{m_p + M_K - m_{\Lambda^\star}} \\
&\quad -\frac{1}{f^2}(m_u + m_s)(a_1 + a_2 + 2a_3) \\
&\quad +\frac{1}{2f^2}(\bar{d}_1 + \bar{d}_3 + 2\bar{d}_5 + \bar{d}_7)M_K^2 \\[2mm]
\mathcal{T}(K^-p \to \bar{K}^0 n) &= \frac{1}{2f^2}M_K - \frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_n)}{m_p + M_K - m_{\Lambda^\star}} \\
&\quad -\frac{1}{2f^2}(m_u + m_d + 2m_s)a_1 \\
&\quad +\frac{1}{2f^2}(\bar{d}_1 + \bar{d}_7)M_K(M_K + m_p - m_n) \\[2mm]
\mathcal{T}(K^-p \to \pi^0\Sigma^0) &= \frac{1}{8f^2}(2M_K + m_p - m_{\Sigma^0}) - \frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_{\Sigma^0})}{m_p + M_K - m_{\Lambda^\star}} \\
&\quad -\frac{1}{4f^2}(3m_u + m_s)a_2 \\
&\quad +\frac{1}{4f^2}(\bar{d}_3 + 2\bar{d}_7 + \bar{d}_9)M_K(M_K + m_p - m_{\Sigma_0}) \\[2mm]
\mathcal{T}(K^-p \to \pi^0\Lambda) &= \frac{\sqrt{3}}{8f^2}(2M_K + m_p - m_\Lambda) \\
&\quad -\frac{1}{4\sqrt{3}f^2}(3m_u + m_s)(-2a_1 + a_2) \\
&\quad +\frac{1}{4\sqrt{3}f^2}(-2\bar{d}_1 + \bar{d}_3 + \bar{d}_9)M_K(M_K + m_p - m_\Lambda) \\[2mm]
\mathcal{T}(K^-p \to \eta\Sigma^0) &= \frac{\sqrt{3}}{8f^2}(2M_K + m_p - m_{\Sigma^0}) \\
&\quad -\frac{1}{4\sqrt{3}f^2}a_2(m_u - 5m_s) \\
&\quad +\frac{1}{4\sqrt{3}f^2}(-\bar{d}_3 + \bar{d}_9)M_K(M_K + m_p - m_{\Sigma^0}) \\[2mm]
\mathcal{T}(K^-p \to \eta\Lambda) &= \frac{3}{8f^2}(2M_K + m_p - m_\Lambda) - \frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_\Lambda)}{m_p + M_K - m_{\Lambda^\star}} \\
&\quad -\frac{1}{12f^2}(2a_1 - a_2)(5m_s - m_u) \\
&\quad +\frac{1}{12f^2}(2\bar{d}_1 - \bar{d}_3 + 6\bar{d}_7 + 5\bar{d}_9)M_K(M_K + m_p - m_\Lambda) \\[2mm]
\mathcal{T}(K^-p \to K^+\Xi^-) &= -\frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_{\Xi^-})}{m_p + M_K - m_{\Lambda^\star}}
\end{aligned}
$$

$$+\frac{1}{2f^2}(\bar{d}_7 + \bar{d}_9)M_K(M_K + m_p - m_{\Xi^-})$$

$$\mathcal{T}(K^-p \to K^0\Xi^0) = -\frac{g_{\Lambda^\star}^2}{f^2}\frac{M_K(M_K + m_p - m_{\Xi^0})}{m_p + M_K - m_{\Lambda^\star}}$$

$$+\frac{1}{2f^2}(\bar{d}_7 + \bar{d}_9)M_K(M_K + m_p - m_{\Xi^0}) \qquad (A.13)$$

where the physical masses of baryon and meson are used. The processes involving $\bar{K}$, η and Ξ do not figure in the branching ratios R_c and R_n[12].

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